Temporal correlations of local network losses

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We introduce a continuum model describing data losses in a single node of a packet-switched network (like the Internet) which preserves the discrete nature of the data loss process. *By construction*, the model has critical behavior with a sharp transition from exponentially small to finite losses with increasing data arrival rate. We show that such a model exhibits strong fluctuations in the loss rate at the critical point and non-Markovian power-law correlations in time, in spite of the Markovian character of the data arrival process. The continuum model allows for rather general incoming data packet distributions and can be naturally generalized to consider the buffer server idleness statistics.

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I. INTRODUCTION

Complex networks underpin many diverse areas of science. They manifest themselves in relationships between network topology and functional organization of complex neuron structures [1,2], interacting organic molecules describing metabolic activity in living cells [3], multispecies food webs [4,5], numerous aspects of social networks [6–9], and the connectivity and operation of the Internet [10–12]. New models of network topology such as scale-free [13] or small world [14] have been found to be surprisingly good at describing real-world structures. A consequence of the realization that complex networks describe universal properties of many such problems has resulted in extensive research activity by the physics community in the past decade (see Refs. [15,16] for reviews).

A problem of particular significance in many application domains is the resiliency of complex networks to the random or selective removal of nodes or links. For example, the loss of connectivity in scale-free networks [10,17–20] has implications on the tolerance of the Internet to protocol or equipment failures. Typically, the site or bond disorder acts as an *input* which makes them very general and applicable to a wide variety of networks.

More recently there has been an increasing realization that network breakdowns can not only result from the physical loss of connectivity, but can arise due to the loss of data traffic in the network (i.e., congestion) [21,22]. However, only a few dynamical models of traffic in networks have been considered to date [11,24–26]. In the case of communication networks the excessive loading of even a single node can give rise to cascades of failures arising from traffic congestion and consequently isolate large parts of the network [27]. To describe the operational failure arising due to congestion at a particular network node, one needs to account for distinct features of the dynamically "random" data traffic which is the reason for such a breakdown.

In this paper we model data losses in a *single node* of a packet-switched network such as the Internet. There are two distinct features which must be preserved in this case: the discrete character of data propagation and the possibility of data overflow in a single node. In the packet-switched network data is divided into packets which are routed from source to destination via a set of interconnected nodes (routers). At each node packets are queued in a memory buffer before being *serviced*, i.e., forwarded to the next node (there are separate buffers for incoming and outgoing packets but we neglect this for the sake of simplicity). Due to the finite capacity of memory buffers and the stochastic nature of data traffic, any buffer can become overflown which results in packets being *discarded*.

We focus on a continuum description of the discrete process of data packet loss. Such a continuum model represents a simplification that preserves the salient features of the data loss mechanism, while at the same time it can be more easily embedded in a larger model describing data packet losses in a large network. The continuum description allows us to overcome inevitable difficulties in incorporating realistic distributions of incoming traffic into a discrete-time class of models, such as one we introduced earlier [23]. On the contrary, the continuum model can easily incorporate a completely general distribution of packet lengths and interarrival times, both essential in modeling data loss in finite-sized buffers.

We introduce a model where noticeable data losses in a single memory buffer start when the average rate of random packet arrivals approaches the service rate. Under this condition the model has *a built-in sharp* transition from free flow to lossy behavior with a sizable fraction of arriving packets being dropped. A sharp onset of network congestion is familiar to everyone using the Internet and was numerically confirmed in different models [28–30]. Here we stress that such a congestion originating from a single node is characterized by strong critical fluctuations of the data loss in the vicinity of the built-in transition.

In particular, we will show that a Markovian input process can give rise to long-range temporal correlations of data losses that are strongly non-Markovian in the critical regime. In the context of the Internet, this means that when excessive data losses start it is more probable that they persist for a while, thus impacting on network operation. As we will discuss later in this paper, this non-Markovian behavior has a profound effect on the operation of current Internet protocols, such as the transport control protocol (TCP), that dictate how users experience the network operation.

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While data loss is natural and inevitable due to data overflow, we show that loss rate statistics turn out to be highly nontrivial in the realistic case of a finite buffer, where at the critical point the magnitude of fluctuations can exceed the average value. The fluctuations still obey the central limit theorem but only in the unrealistically long-time limit. The importance of fluctuations in some intermediate regime is a definitive feature of *mesoscopic* physics, albeit the reasons for this are absolutely different (note that even in the case of electrons, the origin of the mesoscopic phenomena can be either quantum or purely classical, see, e.g., [31]).

The *average* loss rate and/or transport delays were previously studied, e.g., in the theories of bulk queues [32,33] or a jamming transition in traffic flow [34]. What makes present considerations intrinsically different from these theories is the very nature of the quantity we consider: the losses (not existing in flow models) make the description of the traffic process essentially non-Hermitian. Although fluctuations in network dynamics were previously studied (see, e.g., Refs. [11,30,35]), this was done through measurements or numerical simulations of data traffic. Here we present an analytical model of these fluctuations and their temporal correlations.

Due to the symmetry of the continuum description of a buffer with respect to its full (lossy) and empty (idle) states, we also derive corresponding expressions for the statistics of idleness of the buffer server (i.e., output links from routers). This quantity is essential in determining the way the statistics of data traffic going into a subsequent buffer along a data path are shaped. This is self-evidently important when we are attempting to describe the operation of an entire network.

II. THE MODEL

We consider a single finite-size memory buffer fed with a random data-packet stream. It stores the packets and then is *serviced* by the data link that sends this packets further along the network on a first-in–first-out basis. This adequately models the output buffer attached to the switching device in the router. The speed of the input line of the buffer is much bigger than the speed of the output line. The reason is that the input comes from the switching fabric of a router which is designed to operate very fast indeed in order to feed a large number of such buffers, but sequentially. The capacity of the output line is normally smaller.

Hence, we can model the packet arrival as an instantaneous renewal process. The storage capacity of the buffer is L, measured in bits. The lengths of arriving packets are treated as random, all being much smaller than L. The service rate (i.e., the rate at which packets depart from the buffer) is considered to be deterministic, as randomness in it is negligible as compared to that of the input traffic. We normalize the lengths of packets p, the speed of the output link r_{out}, and the queue length ℓ by the size of the buffer L (which is henceforth set to 1).

The procedure for the renewal cycle is described as follows: at the moment of arrival of a packet of size p, the state of the queue is ℓ, this is followed by the gap η (random interarrival time) until the next arrival. We introduce the time scale required to empty a full buffer provided there are no new arrivals, $\eta_0 \equiv 1/r_{\text{out}}$. If $\ell+p \leq 1$ then the packet joins the queue and the queue length prior to the next arrival is $\ell' = \ell + p - \eta/\eta_0$ if $\ell' > 0$ and $\ell' = 0$ otherwise. If $\ell + p > 1$ then the packet is discarded and the queue length prior to the next arrival is $\ell' = \ell - \eta/\eta_0$ if $\ell' > 0$ and $\ell' = 0$ otherwise.

Since the maximum packet size is much less than 1 (the buffer size) and assuming that the average incoming traffic rate r_{in} (also normalized to the buffer size) is close to the service rate

$$|r_{\text{in}}\eta_0 - 1| \ll 1, \tag{1}$$

we can treat p, η, and ℓ as continuous stochastic variables. The packet length, p, and the packet interarrival gap, η, are the input random variables in our theory. We can incorporate any generic distribution of each of these variables into the input stochastic parameters of the model under consideration (provided that all the moments of such a distribution are finite which is always true for any realistic distribution).

Our aim is to calculate the statistics of the amount of the dropped traffic and the service lost due to idleness of the output link during time $t \gg \bar{\eta}$ ($\bar{\eta}$ is the average interarrival time) in the regime (1). In this regime and for observation times $t \gg \bar{\eta}$, the system can be described by the Fokker-Planck equation [36]. We introduce the transitional probability density function, $w(\ell', t; \ell)$, where $w(\ell', t; \ell)d\ell'$ is the conditional probability that the queue has length between ℓ' and $\ell' + d\ell'$ at the time t, provided that it had length ℓ at time 0. Then we have

$$\partial_t w(\ell', t; \ell) = -a\partial_{\ell'} w(\ell', t; \ell) + \frac{1}{2}\sigma^2 \partial^2_{\ell'} w(\ell', t; \ell), \tag{2}$$

where a and σ^2 are the average and the variance of the change of the queue size per unit time

$$a \equiv \frac{1}{\Delta t}\langle \Delta \ell \rangle, \quad \sigma^2 \equiv \frac{1}{\Delta t}\langle \Delta \ell^2 \rangle, \quad \Delta t \to 0. \tag{3}$$

Here $\langle \cdots \rangle$ means the ensemble averaging over random input, i.e., over all the packets lengths p and interarrival times η. Irrespective of the distributions of p and η, only their first two moments determine the statistics (3) provided that the time scale is chosen as described above. Note that in the limit $\Delta t \to 0$ the variance is, indeed, the second moment of the change of the queue size, defined in the above equation. The natural boundary and initial conditions for Eq. (2) are imposed as follows:

$$J(\ell', t; \ell)|_{\ell'=0,1} = 0, \tag{4}$$

$$w(\ell', t; \ell)|_{t=0} = \delta(\ell' - \ell), \tag{5}$$

where

$$J(\ell', t; \ell) \equiv a w(\ell', t; \ell) - \frac{1}{2}\sigma^2 \partial_{\ell'} w(\ell', t; \ell) \tag{6}$$

is the probability current. This means that this current vanishes at the boundary which translates crudely to the intuitively obvious statement that the probability cannot flow beyond the queue boundaries $\ell = 0$ and $\ell = 1$. The condition $\Delta t \to 0$ in Eq. (3) means that Δt is much smaller than the observation time, but still large enough so that the underlying stochastic processes can be considered as continuous [36],

$$\bar{\eta} \ll \Delta t \ll t. \tag{7}$$

The solution of Eqs. (2), (4), and (5) can be formally expressed as follows:

$$w(\ell',t;\ell) = 2e^{v(\ell'-\ell)}\sum_{k=1}^{\infty}\frac{\exp[-(4\pi^2k^2+v^2)\tau]}{4\pi^2k^2+v^2} \times[2\pi k\cos(2\pi k\ell')+v\sin(2\pi k\ell')] \times[2\pi k\cos(2\pi k\ell)+v\sin(2\pi k\ell)], \tag{8}$$

where

$$v \equiv \frac{a}{\sigma^2}, \quad \tau \equiv \frac{\sigma^2 t}{2}. \tag{9}$$

Note that the solution (8) can be expressed in terms of θ functions.

For the Laplace transform of $w(\ell',t;\ell)$ we have

$$\begin{aligned} W(\ell',\epsilon;\ell) &\equiv \mathcal{L}_\tau w(\ell',t;\ell) \\ &= \frac{1}{2}\frac{e^{v(\ell'-\ell)}}{\kappa\sinh(\kappa)}\left\{\frac{2v^2}{\epsilon}\cosh[\kappa(\ell'+\ell-1)]\right. \\ &+\frac{2\kappa v}{\epsilon}\sinh[\kappa(\ell'+\ell-1)] \\ &+\cosh[\kappa(|\ell'-\ell|-1)] \\ &\left.+\cosh[\kappa(\ell'+\ell-1)]\right\}, \end{aligned} \tag{10}$$

where

$$\kappa \equiv \sqrt{\epsilon+v^2} \tag{11}$$

From Eq. (10) we have the Laplace images of the probability densities of returning to the boundaries as

$$W(0,\epsilon;0) = \frac{1}{\epsilon}[\kappa\coth(\kappa)-v],$$

$$W(1,\epsilon;1) = \frac{1}{\epsilon}[\kappa\coth(\kappa)+v]. \tag{12}$$

These will be used in the next section.

III. STATISTICS OF LOSSES

In this section we concentrate on the statistics of the losses due to the buffer overflowing. The corresponding formulas for the statistics of the server idleness can be obtained using transformation $\ell \to 1-\ell, v \to -v$.

First, we estimate the size of fluctuations of the losses on a time scale $t \ll 2/\sigma^2$. In order to do that we consider the dynamics of the system near the boundary $\ell=1$ which is governed by the following transitional probability:

$$\begin{aligned} w_0(\ell',t;\ell) &= \frac{1}{\sqrt{2\pi\sigma^2 t}}\exp\left[-\frac{a(\ell'-\ell)}{\sigma^2}-\frac{a^2t}{2\sigma^2}\right] \\ &\times\left\{\exp\left[-\frac{(\ell'-\ell)^2}{2\sigma^2 t}\right]\right. \\ &\left.+\exp\left[-\frac{(2-\ell'-\ell)^2}{2\sigma^2 t}\right]\right\} \\ &-\frac{a}{\sigma^2}\exp\left[\frac{2a(1-\ell')}{\sigma^2}\right]\operatorname{erfc}\left[\frac{2-\ell'-\ell+at}{\sqrt{2\sigma^2 t}}\right], \end{aligned} \tag{13}$$

which is the solution of Eq. (2) when the boundary $\ell=0$ is sent to $-\infty$. The change in the state of the system during time t can then be represented as follows:

$$\Delta\ell(t) \equiv \ell'-\ell = \Delta\ell_0(t)+\Delta\ell_{\text{loss}}(\ell',t;\ell), \tag{14}$$

where $\Delta\ell_0(t)$ is the change in the state of the system if there was no boundary, its statistics is determined by

$$\langle\Delta\ell_0(t)\rangle = at, \quad \langle[\Delta\ell_0(t)]^2\rangle = \sigma^2 t + \text{o}(t), \tag{15}$$

and $\Delta\ell_{\text{loss}}(\ell',t;\ell)$ is the amount of traffic lost due to buffer overflowing with fixed boundary conditions for the queue state: $\ell(0)=\ell$ and $\ell(t)=\ell'$. The moments of the changes of the queue length, Eq. (14), can be defined as follows:

$$\langle[\Delta\ell(t)]^n\rangle = \int d\ell' d\ell(\ell'-\ell)^n w_0(\ell',t;\ell)p(\ell), \tag{16}$$

where $p(\ell)$ is the stationary distribution of buffer occupancy.

For the first two moments defined by Eq. (16) we find in the limit $t \to 0$

$$\langle\Delta\ell(t)\rangle = at+\frac{\sigma^2 t}{2}p(1), \quad \langle[\Delta\ell(t)]^2\rangle = \sigma^2 t. \tag{17}$$

From Eqs. (14), (15), and (17) we can conclude that

$$\langle\Delta\ell_{\text{loss}}(t)\rangle = \frac{\sigma^2 t}{2}p(1),$$

$$\langle[\Delta\ell_{\text{loss}}(t)]^2\rangle + 2\langle\Delta\ell_0(t)\Delta\ell_{\text{loss}}(t)\rangle = o(t). \tag{18}$$

The first of the relations (18) means that $\Delta\ell_{\text{loss}}(\ell',t;\ell)$ is nonzero only if $\ell',\ell \sim 1$ in the limit $t \to 0$. The second relation means either

$$\langle[\Delta\ell_{\text{loss}}(t)]^2\rangle, \quad \langle\Delta\ell_0(t)\Delta\ell_{\text{loss}}(t)\rangle = o(t) \tag{19}$$

or

$$\Delta\ell_{\text{loss}}(t) = -2\Delta\ell_0(t) + o(\sqrt{t}). \tag{20}$$

The relation (20) does not make sense physically, so in what follows we accept the conditions of Eq. (19) and show that they are consistent with the subsequent calculations.

Next we lift the restriction $t \ll 2/\sigma^2$. To this end we introduce the conditional moments of the quantity $\Delta\ell_{\text{loss}}$ defined in Eq. (14) with the condition that the system was in the state ℓ at the beginning of the observation interval,

$$m_{\rm loss}^{(k)}(t;\ell)=\int d\ell'\langle\Delta\ell_{\rm loss}^{k}(\ell',t;\ell)\rangle. \tag{21}$$

In the continuous limit, the quantity $\Delta\ell_{\rm loss}$ can be represented as a certain stochastic time integral. Then the kth power of this integral could be represented in the time-ordered form as follows:

$$m_{\rm loss}^{(k)}(t;\ell)=k!\,r_{\rm loss}^{k}\prod_{i=1}^{k}\int_0^{t_{i+1}}dt_i\prod_{j=1}^{k-1}w(1,t_{j+1}-t_j;1)\times w(1,t_1;\ell),\quad t_{k+1}\equiv t, \tag{22}$$

where $w(\ell',t;\ell)$ is determined by Eq. (8) and

$$r_{\rm loss}\equiv\lim_{t\to0}\frac{1}{t}\int d\ell'\int d\ell\,\Delta\ell_{\rm loss}(\ell',t;\ell)=\lim_{t\to0}\frac{1}{t}\int_{-\infty}^{1}d\ell'd\ell\,(\ell'-\ell-at)w_0(\ell',t;\ell)=\frac{\sigma^2}{2}. \tag{23}$$

Equation (22) is exact in the continuous limit and can be interpreted as follows. The losses occur only when the system reaches the upper boundary $\ell=1$, i.e., the buffer is full and any extra packet is discarded. The probability density $w(1,t_1;\ell)$ in Eq. (22) describes the first loss event for the system which started in state ℓ while the product describes subsequent consecutive loss events.

For the corresponding unconditional moments of losses in the stationary regime we have

$$m_{\rm loss}^{(k)}(t)\equiv\int_0^1 d\ell\,m_{\rm loss}^{(k)}(t;\ell)p(\ell)=k!\,p(1)\prod_{i=1}^{k}\int_0^{\tau_{i+1}}d\tau_i\prod_{j=1}^{k-1}w(1,t_{j+1}-t_j;1), \tag{24}$$

where τ is defined in Eq. (9) and $p(\ell)$ is the stationary solution of Eq. (2),

$$p(\ell)=\frac{2ve^{2v\ell}}{e^{2v}-1}. \tag{25}$$

To calculate the moments $m_{\rm loss}^{(k)}(t)$ we consider their Laplace transforms,

$$M_{\rm loss}^{(k)}(\epsilon)\equiv\mathcal{L}_\tau m_{\rm loss}^{(k)}(t)=\int_0^\infty d\tau e^{-\epsilon\tau}m_{\rm loss}^{(k)}(2\tau/\sigma^2)=k!\,p(1)[W(1,\epsilon;1)]^{k-1}\frac{1}{\epsilon^2}, \tag{26}$$

where $W(1,\epsilon;1)$ is defined by Eq. (10).

Taking now the inverse Laplace transform we have

$$m_{\rm loss}^{(k)}(t)\equiv\mathcal{L}_\epsilon^{-1}M_{\rm loss}^{(k)}(\epsilon)=\frac{1}{2\pi\mathrm{i}}\int_{\gamma-\mathrm{i}\infty}^{\gamma+\mathrm{i}\infty}d\epsilon e^{\epsilon\tau}M_{\rm loss}^{(k)}(\epsilon). \tag{27}$$

From Eq. (26) we obtain

$$m_{\rm loss}^{(1)}(t)=p(1)\tau=p(1)\frac{\sigma^2 t}{2}. \tag{28}$$

For the Laplace transforms of Eq. (26) with $k>1$ we can identify the following two regimes:

$$M_{\rm loss}^{(k)}(\epsilon)\approx\begin{cases}k!\,p(1)\epsilon^{-(k+3)/2}, & \epsilon\gg1\\ k!\,p^k(1)\epsilon^{-(k+1)}, & \epsilon\ll1.\end{cases} \tag{29}$$

Thus we obtain the moments defined by Eq. (27) as follows:

$$m_{\rm loss}^{(k)}(t)\approx\begin{cases}k!\,p(1)\dfrac{\tau^{(k+1)/2}}{\Gamma[(k+3)/2]}, & \tau\ll1\\ p^k(1)\tau^k, & \tau\gg1.\end{cases} \tag{30}$$

Now we calculate the probability density function (PDF) $p_{\rm loss}(x;t)$ of the amount of the lost traffic, x, during time t. To calculate it we consider its characteristic function in the ϵ representation,

$$\tilde{P}_{\rm loss}(s;\epsilon)\equiv\mathcal{L}_x P_{\rm loss}(x;\epsilon),$$

$$P_{\rm loss}(x;\epsilon)\equiv\mathcal{L}_\tau p_{\rm loss}(x;t). \tag{31}$$

On performing explicitly the Laplace transform in Eq. (31) we obtain the standard expression for the characteristic function in terms of the Laplace transforms of the unconditional moments of losses given by Eq. (26),

$$\tilde{P}_{\rm loss}(s;\epsilon)=\sum_{k=0}^{\infty}\frac{(-s)^k}{k!}\int_0^\infty dx x^k\mathcal{L}_\tau p_{\rm loss}(x;t)=P_{\rm loss}(\epsilon)+\sum_{k=1}^{\infty}\frac{(-s)^k}{k!}M_{\rm loss}^{(k)}(\epsilon), \tag{32}$$

where

$$P_{\rm loss}(\epsilon)=\mathcal{L}_\tau p_{\rm loss}(t),\quad p_{\rm loss}(t)=\int_0^\infty dx p_{\rm loss}(x,t), \tag{33}$$

with $1-p_{\rm loss}(t)$ being the probability for the system not to drop a single packet over the period of time t. Substituting $M_{\rm loss}^{(k)}(\epsilon)$, Eq. (26), into Eq. (32), we find

$$\tilde{P}_{\rm loss}(s;\epsilon)=P_{\rm loss}(\epsilon)+\frac{p(1)}{\epsilon^2}\sum_{k=1}^{\infty}(-s)^k[W(1,\epsilon;1)]^{k-1}=P_{\rm loss}(\epsilon)+\frac{p(1)}{\epsilon^2W(1,\epsilon;1)}\left[-1+\frac{1}{1+sW(1,\epsilon;1)}\right].$$

In order that $P_{\rm loss}(s;\epsilon)$ does not have an abnormal behavior [in particular, it does not contain terms such as $\delta(x)$], we must assume that

$$P_{\rm loss}(\epsilon)=\frac{p(1)}{\epsilon^2W(1,\epsilon;1)}. \tag{34}$$

Hence,

$$P_{\rm loss}(x;\epsilon) = \frac{p(1)}{\epsilon^2 W^2(1,\epsilon;1)} \exp\left[-\frac{x}{W(1,\epsilon;1)}\right]. \quad (35)$$

Integrating this relation over x, we recover Eq. (34), which shows that our assumption is indeed correct.

In the regimes of short and long times we have

$$p_{\rm loss}(x;t) \approx \begin{cases} p(1)\mathrm{erfc}\left[\frac{x}{\sqrt{4\tau}}\right], & \tau \ll 1 \\ \delta[x-\tau p(1)], & \tau \gg 1, \end{cases} \quad (36)$$

and

$$p_{\rm loss}(t) \approx \begin{cases} p(1)\sqrt{\frac{4\tau}{\pi}}, & \tau \ll 1 \\ 1, & \tau \gg 1. \end{cases} \quad (37)$$

The conditional PDF (with the condition that the system dropped at least one packet during the time t) can be defined as follows:

$$w_{\rm loss}(x;t) \equiv \frac{p_{\rm loss}(x;t)}{p_{\rm loss}(t)} \approx \begin{cases} \sqrt{\frac{\pi}{4\tau}}\,\mathrm{erfc}\left[\frac{x}{\sqrt{4\tau}}\right], & \tau \ll 1 \\ \delta[x-\tau p(1)], & \tau \gg 1. \end{cases} \quad (38)$$

It is most important to stress here that the fluctuations of losses are strong for physically long times given by the first line above; however, these times are still short on the scale of $1/\sigma^2$. For the unphysically long time, the second line above, the fluctuations obviously obey the central limit theorem.

Now let us compare the results of the present approach with those of considerations in Ref. [23] where a simple discrete-time model for studying losses in a single buffer was introduced. In that model packets of fixed size arrive with probability p at the equidistant time epochs. The service was deterministic, and half of the packet was served between successive time epochs. To make the comparison, we calculate the central moments of losses in a similar way as the unconditional ones in Eq. (24). Here we will consider the variance of the losses $\sigma^2_{\rm loss}(t)$ only in the limit $\tau \gg 1$,

$$\sigma^2_{\rm loss}(t) \approx m^{(1)}_{\rm loss}(t)\left[\frac{1}{|v|}\coth|v| - \sinh^{-2}|v|\right] \approx \begin{cases} \frac{2}{3}m^{(1)}_{\rm loss}(t), & |v| \ll 1 \\ \frac{1}{|v|}m^{(1)}_{\rm loss}(t), & |v| \gg 1. \end{cases} \quad (39)$$

In this long-time limit the ratio of the variance to the square of the average vanishes, so that the distribution of data losses obeys the central limit theorem, as also seen from the second line of Eq. (38). This is essentially in agreement with the result of the compressibility χ_∞ in [23]. Naturally, the present considerations are much more general as we have not imposed any artificial limitations on the random input traffic.

Finally, we calculate the correlator of the fluctuations of losses measured during two time intervals of length t_1 and t_2 correspondingly and separated by the time T,

$$\mathrm{corr}(t_1,t_2,T) = \int_0^1 d\,\ell\, \rho(t_1,t_2,T) - m^{(1)}_{\rm loss}(t_1)m^{(1)}_{\rm loss}(t_2)$$

where

$$\rho(t_1,t_2,T) = r^2_{\rm loss}\int_0^{t_1} dt_1' \int_0^{t_2} dt_2' w(1,t_1'+t_2-t_2'+T;1)p(1)$$

with $r_{\rm loss}$ defined in Eq. (23).

In the regime $T \gg t_1,t_2$ and $T \gg 2/\sigma^2$ it can be shown that

$$\mathrm{corr}(t_1,t_2,T) \underset{T\to\infty}{\to} 0, \quad (40)$$

as we would expect. In fact, the correlator goes to zero exponentially if $v \neq 0$. In the opposite regime $2/\sigma^2 \gg T \gg t_1,t_2$ we have

$$\mathrm{corr}(t_1,t_2,T) = m^{(1)}_{\rm loss}(t_1)m^{(1)}_{\rm loss}(t_2)\frac{1}{p(1)}\sqrt{\frac{2}{\pi\sigma^2 T}}, \quad (41)$$

which is again in agreement with the results of the discrete-time considerations [23].

IV. DISCUSSION AND CONCLUSION

As we would expect intuitively, loss events separated widely in time are uncorrelated as shown by Eq. (40). By widely separated in time, we mean that the time separation of the two observation intervals in which losses occur is much longer than the time over which fluctuations of queue length become comparable or much bigger than the buffer size itself, i.e., $2/\sigma^2$.

However, in the case when the separation time is much smaller than $2/\sigma^2$, the correlations of loss fluctuations are decaying very, very slowly, as can be seen from Eq. (41). Such time intervals are likely to be comparable or even smaller than the round trip times for typical TCP connections. TCP is the protocol that controls the rate at which data is sent across a network, between a particular source and destination. The exact details of the congestion control operation of TCP can be found in [37]. For our purposes we shall only focus on its salient congestion control features and the implications of the result of Eq. (41) on it.

TCP limits its sending rate as a function of the perceived network congestion. It operates on a virtual control loop of sending packets, receiving acknowledgements, and estimating the round-trip time. Once a packet is lost, the sender cuts its transmission rate by half. If no other loss is detected it increases its sending rate linearly by a small increment. But if a subsequent loss event is detected it cuts its transmission rate in half again. If successive loss events occur, which according to Eq. (41) is likely on the relevant time scale, the reduction in transmission rate can be dramatic and potentially unnecessary. As there are multiple TCP connections experiencing losses at the same buffer this will lead to a cycle of rapid underusage and slow convergence to congestion, which is clearly undesirable and ineffective.

The existence of a phase transition for network losses, as well as strong fluctuations in its vicinity, have been numerically demonstrated in Ref. [28]. Furthermore, the strong temporal fluctuations of losses are in qualitative agreement with the numerical results for the TCP throughput (and loss) in Ref. [29] (see Figs. 3–6 therein).

Studying of spatial correlations of loss fluctuations over a network is work in progress. This will help us quantify the second significant aspect of TCP operation which is its reaction to time-out events, as this is connected to correlated losses and delays around the sequence of buffers forming each control loop.

To conclude, we emphasize that the stability of a network with respect to data loss was mostly analyzed in the past from the viewpoint of the loss of physical connectivity in the network topology where a failure of a given node or link was treated as a (probabilistic) input into a network model. Here we have studied *dynamical* fluctuations in data loss in a single node (memory buffer) of the network. We have shown that the strong fluctuations and long-time memory in losses inevitably follow from the discrete character of signal propagation in the packet-switched networks. This single-node fluctuations can potentially trigger a cascade of failures in neighboring nodes and thus result in a temporal failure of large parts of the network. Naturally, correlations of losses between nodes are important (see, e.g., Ref. [30]) and will be incorporated in an appropriate manner. In the next stage, we intend to utilize these features of the local data loss as dynamical inputs into the network and thus study possible abrupt increase of data loss in the network triggered by a local overload.

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